

If

and

Only if

and

If and only if

Logicians like to translate 'if' (sometimes with an optional 'then'), 'only if', and 'if and only if' in three different ways. Here's a summary of the relevant translations:

If P (then) Q	$P \rightarrow Q$
P if Q	$Q \rightarrow P$
P only if Q	$P \rightarrow Q$
P if and only if Q	$P \leftrightarrow Q$

Why do we do this?

Starting with the easiest case, there's a crucial difference between " P if Q " and "If P , Q ". Think about the meanings of these three sentences:

- 1. If we're in Houston, then we're in Texas.*
- 2. We're in Houston if we're in Texas.*
- 3. We're in Texas if we're in Houston.*

It's obvious (I hope) that sentence 1 means the same thing as sentence 3 but **not** the same thing as sentence 2.

Now, we use $P \rightarrow Q$ as a translation of “If P then Q ”. Since “ Q if P ” means the same thing, we should translate it the same way:

If P (then) Q	$P \rightarrow Q$
Q if P	$P \rightarrow Q$

In each of these sentences, the same sentence comes **after the word ‘if’**. So, in general, when we have a conditional, the sentence immediately after ‘if’ is the **antecedent** of the conditional.

Now, consider the difference between **if** and **only if**. Compare these two sentences:

1. *The patient survived if he had the operation.*

2. *The patient survived only if he had the operation.*

Do these mean the same thing? Compare what happens if we add a little more to each:

1. The patient survived if he had the operation, and even then he didn't survive if the the surgeon was not unusually skillful.*

2. The patient survived only if he had the operation, and even then he didn't survive if the surgeon was not unusually skillful.*

Sentence 1* seems to be self-contradictory: it says that the patient **will survive** if he has the operation, but then it takes that away by saying that under certain circumstances he **will not** survive if he has the operation. Sentence 2*, on the other hand, seems to be perfectly consistent.

But consider these two sentences:

3. **If** *the patient survived*, **then** *he had the operation*.

4. *The patient survived* **only if** *he had the operation*.

Do these mean the same thing? If so, then we should translate them the same way:

The patient survived \rightarrow *he had the operation*.

Finally, consider '**if and only if**'. If 'if' and 'only if' mean different things, then 'if and only if' can't mean the same thing as each of them. In fact, it means the combination of both of them (as you might guess).

*"The patient survived **if and only if** he had the operation"*

=

*"The patient survived **if** he had the operation,*

and

*the patient survived **only if** he had the operation"*

You might think of translating this as follows:

(The patient had the operation \rightarrow he survived)

&

(the patient survived \rightarrow he had the operation)

However, logicians have a special fondness for this kind of sentence, and so they have a special connective for just such a case:

The patient survived \leftrightarrow he had the operation