Predicate Logic

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Using what we've learned so far

Consider the following:

"If borders are secure, then terrorists cannot enter the country. If terrorists cannot enter the country, then acts of terrorism will be reduced. Therefore, if borders are secure, then acts of terrorism will be reduced."

Recognize that it is an argument

- "If borders are secure, then terrorists cannot enter the country. If terrorists cannot enter the country, then acts of terrorism will be reduced. ***Therefore***, if borders are secure, then acts of terrorism will be reduced."

Extract its form

P1 - If borders are secure, then terrorists cannot enter the country.

P2 - If terrorists cannot enter the country, then acts of terrorism will be reduced.

C - If borders are secure, then acts of terrorism will be reduced."

Symbolize the sentences …

P1 - $(S \rightarrow E)$

P2 - $(E \rightarrow R)$

C - $(S \rightarrow R)$

To test for Validity by Proof …

1	(1)	$(S \rightarrow E)$	A
2	(2)	$(E \rightarrow R)$	A
3	(3)	S	A (for $\rightarrow$I)
1,3	(4)	E	1,2 $\rightarrow$E
1,2,3	(5)	R	2,4 $\rightarrow$E
1,2	(6)	$(S \rightarrow R)$	5 $\rightarrow$I (3)

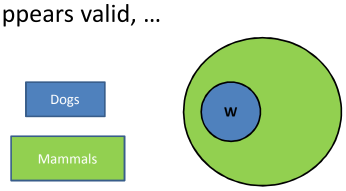
Or by Truth Table

E	R	S	(S → E)	(E → R)	(S → R)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	T	T	T

But what about arguments like this:

All dogs are mammals, and Woofers is a dog.
Therefore Woofers is a mammal.

It appears valid, …



But representing its validity in SL fails

P1 - D (All dogs are mammals.)
P2 - W (Woofers is a dog.)
C - M (Woofers is a mammal.)

D, W ⊢ M is clearly an invalid sequent

What's the problem?

- The problem is that some arguments that are valid cannot be represented as such because their validity stems from the internal structure of atomic sentences, and from the perspective of Sentential Logic, that is hidden information. To represent these types of arguments we need a formal language that is robust enough to represent the internal structure of atoms.

How do atomic sentences work?

In general, simple sentences work by identifying a *subject* and stating something about that subject.

For example:

Spot + _____ is a dog
Woofers + _____ is a dog
Reveille + _____ is a dog

Vocabulary of Predicate Logic

- Sentence Letters
- Connectives
- Names
- Variables
- Predicate Letters
- Quantifiers
- Parentheses

(Our text mentions the Identity Symbol, but we won't be using that in this class.)

Names

- To pick out specific subjects, we use names, e.g. Bob, Jane, etc., and represent them with lower-case letters from the start of the alphabet:

  a, b, c, d, e, … a_1, b_1, c_1, d_1, e_1, …

Variables

- In English we often have non-specific subjects, e.g. "someone," "something," "everyone," etc. In order to help us represent these subjects, we need the help of variables, for which we will use lower-case letters from the latter half of the alphabet:

  u, v, w, x, y, z, … u_1, v_1, w_1, x_1, y_1, z_1, …

1-place Predicate Letter

- A **1-PLACE PREDICATE LETTER** is any symbol from the following list: A^1, B^1, C^1,…, A^1_1, B^1_1, C^1_1, A^1_2, B^1_2, C^1_2,…

  We use these to represent the portion of a sentence where a singular property is ascribed to a subject, e.g.

  ________ is a dog
  ________ is blue
  ________ is an Aggie

2-place Predicate letter

- A **2-PLACE PREDICATE LETTER** is any symbol from the following list: A^2, B^2, C^2,..., A^2_1, B^2_1, C^2_1, A^2_2, B^2_2, C^2_2,...

  In general, we use these to represent the portion of a sentence where a relational property is ascribed, e.g.

  _____ is next to ______
  _____ loves _______
  _____ hates _______

N-place Predicate Letter

- In general, an ***n*-PLACE PREDICATE LETTER** is any symbol from the list: A^n, B^n, C^n,..., A^n_1, B^n_1, C^n_1, A^n_2, B^n_2, C^n_2,...

  For more complex relations, we can create many-place predicate letters to represent things such as:

  ______ is between _______ and _______

Universal Quantifier

- When dealing with unspecific subjects, we usually need to reference quantity, e.g. "every," for that we use the Universal Quantifier which we symbolize as:

$$\forall\alpha$$

(Where α is a variable)

Existential Quantifier

- When dealing with unspecific subjects, we usually need to reference quantity, e.g. "some," for that we use the Existential Quantifier which we symbolize as:

$$\exists\alpha$$

(Where α is a variable)

Expression

- An **EXPRESSION OF PREDICATE LOGIC** is any sequence of items from the vocabulary of predicate logic.

Well-Formed Formulas

- A WELL-FORMED FORMULA of predicate logic is any expression in accordance with the following six rules:

(1) Sentence letters are wffs.
(2) An n-place predicate letter followed by n names is a wff.
(3) Negations, conjunctions, disjunctions, conditionals, and biconditionals of wffs are wffs.
(The formation rules of chapter 1 are subsumed by this clause.)

Universal WFF

(4) If ϕ is a WFF, then the result of replacing at least one occurrence of a name in ϕ by a new variable α (i.e., α not in ϕ) and prefixing $\forall\alpha$ is a WFF.

(Such WFFs are called "universally quantified," or just "universals.")

Existential WFF

(5) If ϕ is a WFF, then the result of replacing at least one occurrence of a name in ϕ by a new variable α (i.e., α not in ϕ) and prefixing $\exists\alpha$ is a WFF.

(Such WFFs are called "existentially quantified," or just "existentials.")

Nothing Else

- Nothing Else