PHIL 240, Introduction to Logic, Sections 501-509 Fall 2014
EXAM 2: Practice

Name (5 points): ________________________________ **Section** (5 points): _______

Section I True / False questions (5 points each)

1. _TRUE__ Only universal and existential WFFs have instances.
2. _FALSE_ A WFF in predicate logic may contain a free variable.
3. _FALSE_ If a WFF begins with the symbols "∀x", then it must be an existential.
4. _FALSE_ All valid arguments have a countermodel.

Section II Mark the correct completion (5 points each)

1. The condition on ∀I requires that …

   (a) _____ the instantial name must occur in at least one of the sentences in the assumption of the line to which one applies the rule.

   (b) _____ there is no condition on the application of ∀I.

   (c) __X__ the instantial name cannot occur in any sentence in the assumption set of the line to which one applies the rule.

   (d) _____ a free variable must be used in place of an instantial name.

   (e) _____ the instantial name be used in the sentence which results from the application of the rule.

2. The sentence ∀x(Fx → ~ (∃yGy & R)) is a …

   (a) _____ existential

   (b) _____ conditional

   (c) _____ negation

   (d) __X__ universal

   (e) _____ conjunction

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3. The following is NOT a condition on the application of ∃E …

(a) _____ the instantial name cannot occur in the line that motivates the assumption to be discharged.

(b) _____ the instantial name cannot occur in the line containing the sentence which is repeated.

(c) __X__ the instantial name must occur in the line which is repeated.

(d) _____ the instantial name cannot occur in the assumption set of the line containing the sentence which is repeated save for the assumption itself.

4. A finite interpretation may contain all but …

(a) _____ a universe

(b) _____ predicate extensions

(c) _____ truth value specifications

(d) __X__ a proof

Section III Translations (5 points each)

Using the following translation scheme, construct a strictly correct translations that includes all parentheses.

Bx = 'x is a book' Hx = 'x is a hardback'

Px = 'x is a paperback' Ex = 'x exists'

Lxy = 'x is longer than y'

a = Logic Primer b = 'Crime and Punishment'

1) Among books, only paperback and hardback exist.

$\forall x(Bx \rightarrow (Ex \rightarrow (Hx \lor Px)))$

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2) All books are paperbacks.

∀x(Bx → Px)

3) Crime and Punishment is longer than the Logic Primer, only if Crime and Punishment is a hardback.

(Lba → Hb)

4) Not all books are hardback if paperbacks exist.

(∃x(Px & Ex) → ~∀x(Bx →Hx))

Section IV Proofs (8 points each)

Give a proof for each of the following sequents. You may use both primitive and derived rules.

1. ∀x(Fx v Gx), ∀x(Gx → Hx), ∃x~Fx ├ ∃xHx

1	(1)	∀x(Fx v Gx)	A	
2	(2)	∀x(Gx → Hx)	A	
3	(3)	∃x~Fx	A	├ ∃xHx
4	(4)	~Fa	A (for ∃E on 3)	
1	(5)	(Fa v Ga)	1 ∀E	
1,4	(6)	Ga	4, 5 vE	
2	(7)	(Ga → Ha)	2 ∀E	
1,2,4	(8)	Ha	6, 7 →E	
1,2,4	(9)	∃xHx	8 ∃I	
1,2,3	(10)	∃xHx	3,9 ∃E(4)	

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2. ∀x(Px → (Qx & Rx)), ∃xPx → ∀x~Rx ├ ~∃xPx

1	(1)	∀x(Px → (Qx & Rx))	A	
2	(2)	∃xPx → ∀x~Rx	A	├ ~∃xPx
3	(3)	∃xPx	A (for RAA)	
2,3	(4)	∀x~Rx	2,3 →E	
5	(5)	Pa	A (for ∃E on 3)	
1	(6)	(Pa → (Qa & Ra))	1 ∀E	
1,5	(7)	(Qa & Ra)	5, 6 →E	
1,5	(8)	Ra	7 &E	
2,3	(9)	~Ra	4 ∀E	
1,2,5	(10)	~∃xPx	8, 9 RAA (3)	
1,2,3	(11)	~∃xPx	3, 10 ∃E (5)	
1,2	(12)	~∃xPx	3, 11 RAA (3)	

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Section V Finite Interpretations (2 points each)

For each of the sentences below, indicate whether it is true or false in this finite interpretation:

U: {a, b, c}
F: {a}
G: {a, b, c}
H: {<a,b>, <b,b>}

1. __TRUE_ (Hba → ~ Gb)
2. _FALSE_ ∃x(Fx& ~ Gx)
3. _FALSE_ (∀xGx → ∀xFx)
4. _FALSE_ ~∃xHxx

Section VI Finite Countermodels (6 points)

Construct a counter-model for the following sequent. Be sure to show your work.

∃x(Px & Rx), ∃x(Sx & Rx) ⊢ ∃x(Px & Sx)

U = {a,b}
P = {b}
R = {a,b}
S = {a}

(Pa & Ra) v (Pb & Rb), (Sa & Ra) v (Sb & Rb) ⊢ (Pa & Sa) v (Pb & Sb)
F T T T T T F T F T T F
F T T F F F
T T F